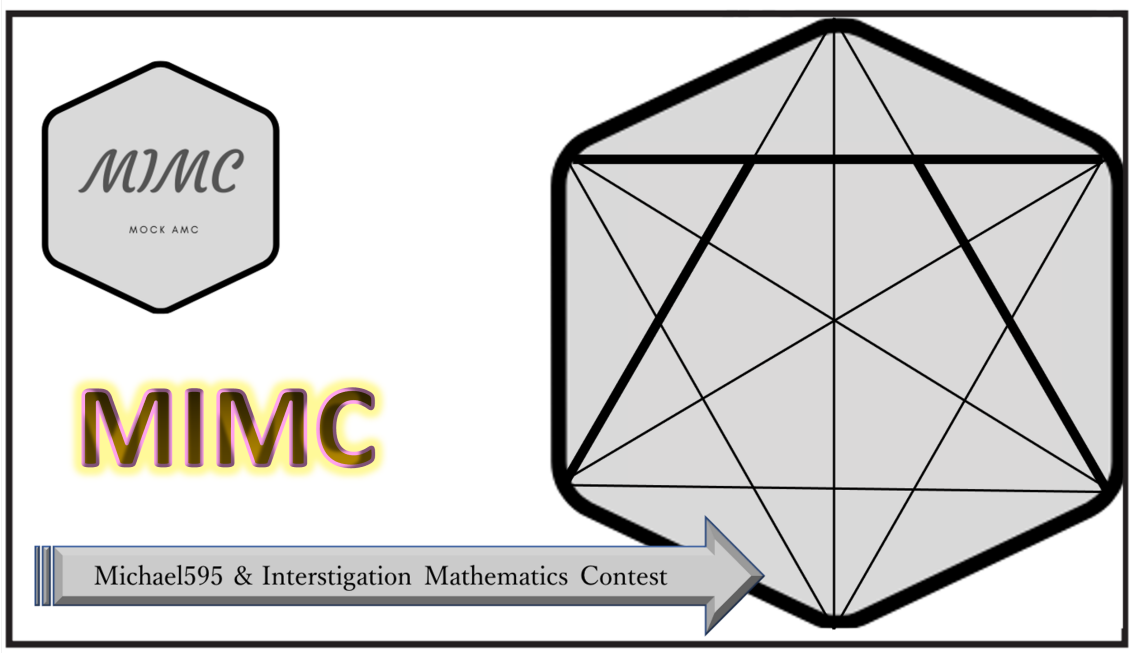

2023 Spring MIMC Contest!

Please read the instructions carefully before you begin.

INSTRUCTIONS

1. DO NOT SCROLL DOWN TO THE PROBLEMS UNTIL YOU ARE READY.
2. This is a ten question short answer test. Each answer is unique and is not limited to integers.
3. You **must** simplify your answers to the simplest possible – that includes rationalizing the denominator, reducing a fraction, etc. You will **not** receive credit for an unsimplified answer.
4. SCORING: You will receive 1 point for each correct answer and 0 points for each incorrect or blank answer.
5. No aids are permitted other than scratch paper, rulers, compass, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When you are ready to start the test, you can begin working on the problems. You will have 30 minutes to complete the test.
8. This is a qualification test, and that being said, every participant with score higher than the threshold would qualify to round 1.
9. When you finish the exam, fill in and submit the Google Form.
10. Enjoy the problems!

The MIMC Committee reserves the right to disqualify scores from an individual if it determines that the required security procedures were not followed.

1. If $a \oplus b = a^2 + b^7$ for all positive integer a and b, find the value of $3 \oplus 4$.
2. Find the area enclosed by $3x^2 + y^2 = 16$.
3. Given that $\frac{a+\sqrt{3}}{a-\sqrt{3}} = 3$ and $\frac{a+k\sqrt{3}}{a-k\sqrt{3}} = 6$, find the value of k.
4. Given that the regular octagon $ABCDEFGH$ has an area of 64, what is the length of BE?
5. How many of the following solids can fit completely inside a unit cube?
   (a) A cube with side length 0.9999,
   (b) A triangular prism with base being an equilateral triangle with side length 1.7 and height 0.001,
   (c) A tetrahedral with side length 1.3,
   (d) A cylinder with base radius of 0.001 and height 1.7.
6. What is the value of $2^2 + 3^2 + 4^2 + ... + 10^2$?
7. Let $f(x) = -\frac{x^2-2x-1}{1-x^2} + 2x\sqrt{1-x^2}$. How many values of x are there between -1 and 1 such that $f(x) = 1$?
8. Farad and Newton are playing a game. Pascal flips a coin records the result after each flip until either player wins. If at any point, the record looks like "head, head, tails", then Farad wins. If the record looks like "tails, head, head", the Newton wins. If each time Farad wins, he gets \$100, and each time Newton wins, Farad gets nothing. Then how much money would Farad be willing to pay to play this game to make it fair?
9. The equation of a parabola with directrix $y = 5x + 3$ and focus at $(2, 15)$ can be expressed as
$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$
find the ordered sextuple (A, B, C, D, E, F).
10. How many ways are there to rearrange $1, 2, 3, 4, ..., 12, 13$ such that for all positive integer a in the list, a appears before $2a$ and $2a$ appears before $3a$?

ADDITIONAL INFORMATION

1. **The Committee on the Michael595 & Interstigation Math Contest (MIMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The MIMC also reserves the right to disqualify score from a test taker if it is determined that the required security procedures were not followed.**
2. **The publication, reproduction or communication of the problems or solutions of the MIMC 10 will result in disqualification. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules except the private discussion form.**

Sincerely, the MIMC mock contest cannot come true without the contributions from the following testsolvers, problem writers and advisors:

Michael595 (Problem Writer)

Interstigation (Problem Writer)